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ORIGINAL ARTICLES

A new formulation for minimum input volt-ampere (VA)-slip relationship of three-phase induction motors

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Abstract The magnetization component of input current of low-rating three-phase induction motor is large which results in poor power factor particularly at low-load conditions. The situation becomes critical when motor is fed from a distributed generating (DG) system. In this paper, the performance analysis of induction motor is carried out for minimum input VA. A distinct novel slip is found at which the input VA is minimum independent of the load conditions. This optimum slip is expressed in terms of motor per-phase equivalent circuit parameters. This novel relationship is valid for every induction motor. As compared to rated conditions, a drastic reduction in VA with a large improvement in power factor is observed. Although, a marginal change in output power and efficiency is recorded. All the analytical, simulated and experimental results match each other with a very fair degree of accuracy.

1. Introduction

The main reason for low power factor in electric power system is induction motors operating below the rated capacity. Therefore, the input current required to deliver the same active power is larger than that for high power factor condition (Krause et al., 2004; Mohan, 2002). The magnetization component of input current of low-rating three-phase induction motors (less than 3-hp) is large which results in poor power factor at light-load conditions. So, a very large VA is drawn by motor from the source. When the motor is fed from a weak power source e.g. distributed generating (DG) system, the situation becomes critical due to its limited VA capacity and becomes more serious in case of faults (Sher et al., 2015). Effective utilization of system resources and the capital cost are largely decided by VA rating of the system. If the VA demand is reduced, system resources will be utilized effectively and further leading to overall reduction of system cost. Hence, VA capacity is more crucial for a DG system than real power input (kW).

The performance of medium/high power induction motors can be optimized by using thyristor based regulators as

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suggested in literature (Zenginobuz et al., 2004). Several conditions for minimum input power, minimum input current and maximum efficiency in terms of machine parameters, slip frequency and line voltage have been proposed in the literature (Vorel et al., 2010). It has been reported that efficiency can be improved by operating the motor at optimum flux (Nasiruddin and Sang Woo, 2008; Waheedabeevi et al., 2012). Voltage and frequency control based optimal efficiency operation has also been suggested (Chen and Yeh, 1992; Creighton et al., 1979; Famouri and Cathey, 1991). Efficiency is improved when input voltage is reduced for low-torque load. But these techniques are suggested only for low-load conditions with no specific limits (Mohan, 1980; Jian et al., 1983). Maximum efficiency occurs at optimum voltage which is variable and dependent on load (Jian et al., 1983). However, no relationship or condition is available for minimizing or optimizing the input VA of induction motors.

Earlier, it was suggested by authors that there exist a unique speed for minimum input VA operation. However, no analytical relationship was proposed for it (Ashfaq and Asghar, 2005). In this paper, a new formulation for minimum input volt-ampere (VA)-slip relationship of three-phase induction motor is proposed. Three-phase induction motor performance analysis is done for minimum input VA, and a distinct novel slip is found in terms of machine per-phase equivalent circuit parameters when VA is minimum and independent of load conditions.

2. Methodology

The parameters of practical machine can be determined by a combination of dc resistance, no load and locked rotor test. From the per-phase equivalent circuit diagram of a three-phase induction motor shown in Fig. 1, the per-phase input volt-amperes (VA_{in}) is given by,

$$\mathrm{VA}_{\mathrm{in}} = \mathrm{VI}_{\mathrm{s}} = I_{\mathrm{s}}^2 \quad \text{(input impedance)}$$

or

$$\mathrm{VA}_{\mathrm{in}} = I_{\mathrm{s}}^2\,(Z_{\mathrm{s}} + \text{equivalent impedance of } Z_{\mathrm{r}} \text{ and } Z_{\mathrm{m}} \text{ in parallel})$$
$$= I_{\mathrm{s}}^2 Z_{\mathrm{s}} + I_{\mathrm{s}}^2\left(\frac{Z_{\mathrm{r}} Z_{\mathrm{m}}}{Z_{\mathrm{r}} + Z_{\mathrm{m}}}\right) \tag{1}$$

where Z_s = stator impedance = $R_s + jX_s$, Z_r = rotor impedance referred to stator = $R_r/s + jX_r$, Z_m = magnetizing circuit impedance, I_s = stator current, I_r = rotor current referred to stator, I_m = magnetizing current.

Also, from Fig. 1,

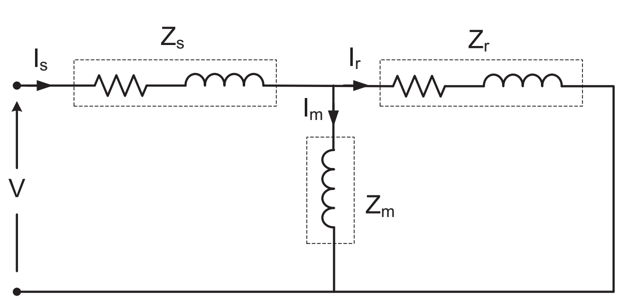

Figure 1 Per-phase equivalent circuit diagram of a three-phase induction motor.

$$I_{\mathrm{s}} = \frac{I_{\mathrm{r}}(Z_{\mathrm{m}} + Z_{\mathrm{r}})}{Z_{\mathrm{m}}} \tag{2}$$

From (1) and (2), the input VA is given by

$$\mathrm{VA}_{\mathrm{in}} = I_{\mathrm{s}}^2 Z_{\mathrm{s}} + I_{\mathrm{r}}^2 \frac{(Z_{\mathrm{m}} + Z_{\mathrm{r}})^2}{Z_{\mathrm{m}}^2} \frac{Z_{\mathrm{r}} Z_{\mathrm{m}}}{(Z_{\mathrm{r}} + Z_{\mathrm{m}})} \tag{3}$$

$$\mathrm{VA}_{\mathrm{in}} = I_{\mathrm{r}}^2\left[Z_{\mathrm{s}}\left(1 + \frac{2Z_{\mathrm{r}}}{Z_{\mathrm{m}}} + \frac{Z_{\mathrm{r}}^2}{Z_{\mathrm{m}}^2}\right)\right] + I_{\mathrm{r}}^2\left[Z_{\mathrm{r}} + \frac{Z_{\mathrm{r}}^2}{Z_{\mathrm{m}}}\right]$$

Since Z_m is very large in comparison to Z_s or Z_r, hence the term Z_r^2/Z_m^2 can be neglected, therefore

$$\mathrm{VA}_{\mathrm{in}} = I_{\mathrm{r}}^2\left[Z_{\mathrm{r}} + Z_{\mathrm{s}}\left(1 + \frac{2Z_{\mathrm{r}}}{Z_{\mathrm{m}}}\right) + \frac{Z_{\mathrm{r}}^2}{Z_{\mathrm{m}}}\right] \tag{4}$$

Since, the per-phase air–gap power of the motor, $P_g = I_r^2 R_r/s$, hence

$$I_{\mathrm{r}}^2 = \frac{sP_{\mathrm{g}}}{R_{\mathrm{r}}} \tag{5}$$

From (4) and (5), the input VA of the motor is given by

$$\mathrm{VA}_{\mathrm{in}} = \frac{sP_{\mathrm{g}}}{R_{\mathrm{r}}}\left\{Z_{\mathrm{s}} + \frac{R_{\mathrm{r}}}{s}\left(1 + \frac{s^2 X_{\mathrm{r}}^2}{R_{\mathrm{r}}^2}\right)^{\frac{1}{2}}\left(1 + \frac{2Z_{\mathrm{s}}}{Z_{\mathrm{m}}}\right) + \frac{X_{\mathrm{r}}^2}{Z_{\mathrm{m}}} + \frac{R_{\mathrm{r}}^2}{s^2 Z_{\mathrm{m}}}\right\} \tag{6}$$

For low-rating motors, R_r and X_r are comparable, hence $(sX_r/R_r) < 1$. From Binomial theorem, $(1 + x)^n \approx 1 + nx$, for $x < 1$. Let $n = 1/2$, then (6) becomes

$$\mathrm{VA}_{\mathrm{in}} = \frac{sP_{\mathrm{g}}}{R_{\mathrm{r}}}\left\{Z_{\mathrm{s}} + \left(\frac{R_{\mathrm{r}}}{s} + \frac{sX_r^2}{2R_{\mathrm{r}}}\right)\left(1 + \frac{2Z_{\mathrm{s}}}{Z_m}\right) + \frac{X_{\mathrm{r}}^2}{Z_{\mathrm{m}}} + \frac{R_{\mathrm{r}}^2}{s^2 Z_{\mathrm{m}}}\right\} \tag{7}$$

When, VA_{in} is minimum, $d(VA_{in})/ds = 0$. Therefore, from (7),

$$\frac{d}{ds}\left[P_{\mathrm{g}}\left\{Z_{\mathrm{s}} s + \left(R_{\mathrm{r}} + \frac{s^2 X_{\mathrm{r}}^2}{2R_{\mathrm{r}}}\right)\left(1 + \frac{2Z_{\mathrm{s}}}{Z_{\mathrm{m}}}\right) + \frac{sX_{\mathrm{r}}^2}{Z_{\mathrm{m}}} + \frac{R_{\mathrm{r}}^2}{sZ_{\mathrm{m}}}\right\}\right] = 0$$

Since, the air–gap power, P_g depends on the load torque. Therefore, for constant-torque loads, P_g remains constant as rotational losses are considered negligible. Thus,

$$\frac{X_{\mathrm{r}}^2}{R_{\mathrm{r}}}\left(1 + \frac{2Z_{\mathrm{s}}}{Z_{\mathrm{m}}}\right)s^3 + \left(\frac{X_{\mathrm{r}}^2}{Z_{\mathrm{m}}} + Z_{\mathrm{s}}\right)s^2 - \frac{R_{\mathrm{r}}^2}{Z_{\mathrm{m}}} = 0 \tag{8}$$

Eq. (8) is used to find the value of slip for which VA is minimum, as the derivative of (8) is always positive in the operating (motoring) region i.e., $0 < s < 1$. It can be seen that the slip for which input VA is minimum is dependent only on machine parameters and is independent on load.

3. Testing of theory and discussion

Several three-phase induction motors are examined experimentally as well as analytically. Fig. 2 shows variation of the input VA with speed for different constant load conditions of a three-phase 1.5 kW induction motor. As proposed from (8), it is observed that the slip for which input VA is minimum is independent of load. Also, an appreciable reduction in input VA is observed for light-load conditions. Even for loads near

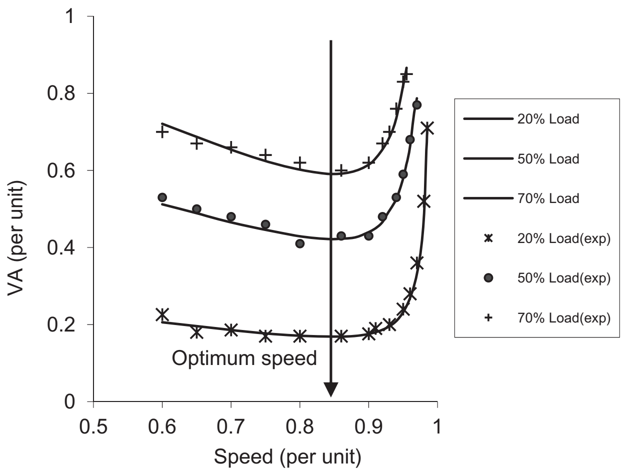

Figure 2 Variation of input VA with speed for different constant-load conditions for a 1.5 kW motor.

to rated, an appreciable reduction in input VA is observed when operated at minimum VA slip.

After testing several low-rating three-phase induction motors, as reported earlier by the authors (Ashfaq and Asghar, 2005), it can be seen from Figs. 3 and 4 that a marginal reduction in the mechanical output power (P_m) occurs at minimum VA operation due to a negligible increase in the slip, and no considerable reduction in efficiency is observed at this condition. Also, simulated and experimental values of VA curve are fairly close to each other.

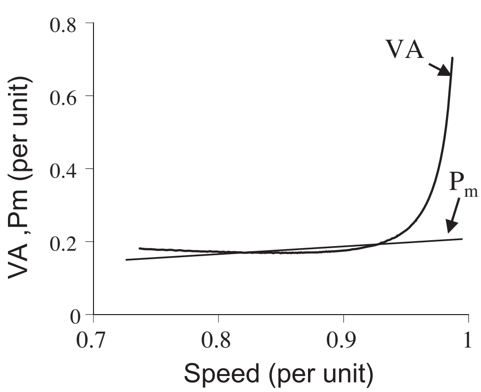

Figure 3 Variation of input VA and P_m with speed at 0.2 pu load of a 1.5 kW motor.

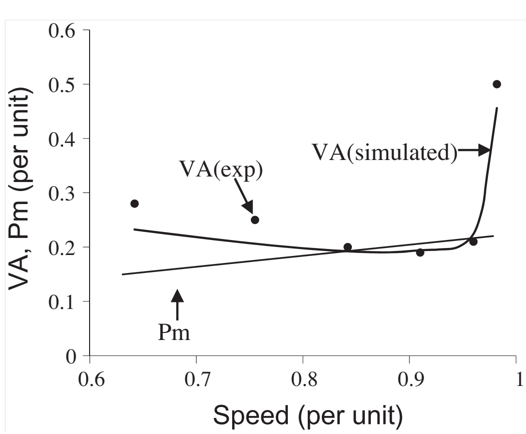

Figure 4 Variation of input VA and P_m with speed at 0.2 pu load of a 2.2 kW motor.

Due to limited VA capacity of the DG systems, only small three phase induction motors are used with it. Small three-phase induction motors draw a large VA from the source which reduces the power factor of the system. Therefore, as the above study shows, operating small motors at minimum VA increases the power factor and helps in reducing the cost of the overall DG system.

4. Conclusion

Performance analysis of three-phase induction motor is done for minimum input VA. A unique novel slip is found in terms of machine equivalent circuit parameters at which the input VA is minimum. This unique novel slip is independent of load and valid for all induction motors. As compared to rated conditions, a drastic reduction in VA with a large improvement in power factor is observed with a marginal change in output power and efficiency. As VA rating decides effective utilization and capital cost of the system, the proposed theory optimizes the use of three phase induction motors with DG systems when the motor is operated at minimum VA slip especially at light-loads. All the analytical, simulated and experimental results match each other with a very fair degree of accuracy.

Appendix A

Machine-1	
Three-phase, Y-connected, 1.5 kW, 50 Hz, 415 V, 3.4 A, 1400 rpm	
Motor parameters:	
Stator resistance, R_s	5.50Ω
Stator reactance, X_s	5.50Ω
Rotor resistance referred to stator side, R'_r	5.93Ω
Rotor reactance referred to stator side, X'_r	5.50Ω
Magnetizing reactance, X_m	84.21Ω
Machine-2	
Three-phase, Y-connected, 2.2 kW, 50 Hz, 400 V, 5.7 A, 925 rpm	
Motor parameters:	
Stator resistance, R_s	2.20Ω
Stator reactance, X_s	3.05Ω
Rotor resistance referred to stator side, R'_r	3.50Ω
Rotor reactance referred to stator side, X'_r	4.50Ω
Magnetizing reactance, X_m	70.10Ω

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